3.4 (25) Find the equation of the tangent line to the graph of y=tan(x) at

a) $x=0$

b) $x=\frac{\pi}{4}$

c) $x=\frac{-\pi}{4}$

$y=\tan(0)=\frac{0}{1}$ $y'=\sec^2(x)$ $y'=1$

$\frac{1}{\cos^2(0)}=1$

$y=1x+0$

$y=\tan\left(\frac{\pi}{4}\right)=1$ $y'=\sec^2(x)$

$\frac{1}{\cos^2\frac{\pi}{4}}=\frac{1}{\frac{\sqrt{2}}{2}}\left(\frac{2}{\sqrt{2}}\right)^2=2$

$y=2x+$

$y=2x+b$

$y=\frac{2\pi}{4}+b$

$1=\frac{\pi}{2}+b$

$-\frac{\pi}{2}$ $-\frac{\pi}{2}$

$b=1-\frac{\pi}{2}$

$y=2x+\left(1-\frac{\pi}{2}\right)$

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Find $g'(4)$ given that $f(4)=3$ and $f'(4)=-5$.

a) $g(x)=\sqrt{x}\,f(x)$

$$g'(x)=\frac{d}{dx}\left(\sqrt{x}\right)\cdot f(x)+\left(\sqrt{x}\right)f'(x)$$

$$g'(x)=\frac{d}{dx}\left(x^{\frac{1}{2}}\right)f(x)+\sqrt{x}\,f'(x)$$

$$g'(x)=\left(\frac{1}{2}x^{-\frac{1}{2}}\right)f(x)+\sqrt{x}\,f'(x)$$

$$g'(x)=\frac{1}{2\sqrt{x}}f(x)+\sqrt{x}\,f'(x)$$

$$g'((4))=\frac{1}{2\sqrt{(4)}}f((4))+\sqrt{(4)}\,f'((4))$$

$$=\frac{1}{4}(3)+(2)(-5)$$

$$=\frac{3}{4}-10=\frac{3}{4}-\frac{40}{4}=\frac{-37}{4}$$

$$(fg)'=f'g+fg'$$

b) $g(x) = \frac{f(x)}{x}$

$$g'(x) = \frac{(f'(x))(x) - (f(x))(1)}{(x)^2}$$

$$g'(4) = \frac{4f'(4) - f(4)}{16}$$

$$= \frac{4(-5) - 3}{16} = \frac{-23}{16}$$

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

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3.3/38)

find $\frac{d}{d\lambda}\left(\frac{\lambda\lambda_0+\lambda^6}{2-\lambda_0}\right)$

$= \frac{d}{d\lambda}\left(\left(\frac{1}{2-\lambda_0}\right)\left(\lambda\lambda_0+\lambda^6\right)\right)$

$= \frac{1}{2-\lambda_0}\frac{d}{dx}\left(\lambda\lambda_0+\lambda^6\right)$

$= \frac{1}{2-\lambda_0}\left[\frac{d}{d\lambda}(\lambda\lambda_0)+\frac{d}{d\lambda}(\lambda^6)\right]$

$= \frac{1}{2-\lambda_0}\left[\lambda_0\frac{d}{d\lambda}(\lambda)+\frac{d}{d\lambda}(\lambda^6)\right]$

$= \frac{1}{2-\lambda_0}\left[\lambda_0(1)+6\lambda^5\right]$

$\frac{d}{dx}(2-\lambda_0)=0$

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$\frac{d}{dx}(cf(x))$

$= c\frac{d}{dx}(f(x))$

$\lim_{x\to ?}(cf(x))$

$= c\lim_{x\to ?}f(x)$

3.5/ Chain Rule

1) first we have to understand to which situations this rule applies

→ product rule applies to products of f^n's

→ quotient rule applies to quotients of f^n's

→ chain rule applies to CHAINS of f^n's

[[commonly known to us as COMPOSITION]]

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Composition of Functions

How would I write

$$x \longmapsto \underset{\text{inside } f^n}{x^2+1} \longmapsto \underset{\text{outside } f^n}{x^3}\ ?$$

$$y = \left(\left(x^2+1\right)\right)^3 = (\underbrace{x^2+1}_{\text{inside}})^{3 \leftarrow \text{outside}}$$

More examples

$$x \mapsto x^2+1 \mapsto \sin x \mapsto x^2$$

$$y = \left(\sin(x^2+1)\right)^2$$

$$x \mapsto 2x \mapsto \tan x \mapsto e^x$$

$$y = e^{\left(\tan(2x)\right)}$$

Chain Rule

Let $y = f(g(x))$

$$\frac{dy}{dx} = y' = f'(g(x)) \cdot g'(x)$$

$$\frac{d}{dx}f(g(x)) = f'(g(x))\cdot g'(x)$$

1) $\frac{d}{dx}(x^2+1)^{75}$

$x \mapsto x^2+1 \mapsto x^{75}$

$75(x^2+1)^{74}(2x) = 150x(x^2+1)^{74}$

2) $\frac{d}{dx}\sin\left(3x^4-\frac{2}{x^2}\right)$

$x \mapsto (3x^4-2x^{-2}) \mapsto \sin x$

$\left[\cos\left(3x^4-\frac{2}{x^2}\right)\right]\cdot(12x^3+4x^{-3})$

$\left(12x^3+\frac{4}{x^3}\right)$

3) $\frac{d}{dx}\sqrt{\sin(\cos x)} = \frac{d}{dx}\left(\sin(\cos x)\right)^{1/2}$

$x \mapsto \cos x$

$\mapsto \sin x$

$\mapsto \sqrt{x}$

$\frac{1}{2}\left(\sin(\cos x)\right)^{-\frac{1}{2}}\cdot\frac{d}{dx}\left(\sin(\cos x)\right)$

$x \mapsto \cos x$

$\mapsto \sin x$

$\frac{1}{2}\left(\sin(\cos(x))\right)^{-\frac{1}{2}}\left(\cos(\cos x)\cdot\frac{d}{dx}(\cos(x))\right)$

$\frac{1}{2}\left(\sin(\cos x)\right)^{-\frac{1}{2}}\left(\cos(\cos x)\right)(-\sin x)$

3.5/1

We KNOW that

$f'(0) = 2$

$g(0) = 0$

$g'(0) = 3$

mult: $\cdot$

composition: $\circ$

$$(f \circ g)'(0) = \left(f(g(x))\right)'\Big|_{x=0}$$

$$f'(g(0)) \cdot g'(0)$$

$$f'(0) \cdot (3) = (2)(3) = 6$$

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